ساخت توابع چندخطی با استفاده از مبهم‌سازی

پویا فرشیم

(کار مشترک با مارتین آلبرشت، دنیس هوف‌هاینز، انریکو لارایا و کنی پترسون)

دانشگاه کوئینز بلفاست – اکول نرمال سوپریور

جنبه‌های ریاضی علوم کامپیوتری – مبانی رمز

Multilinear Maps from Obfuscation

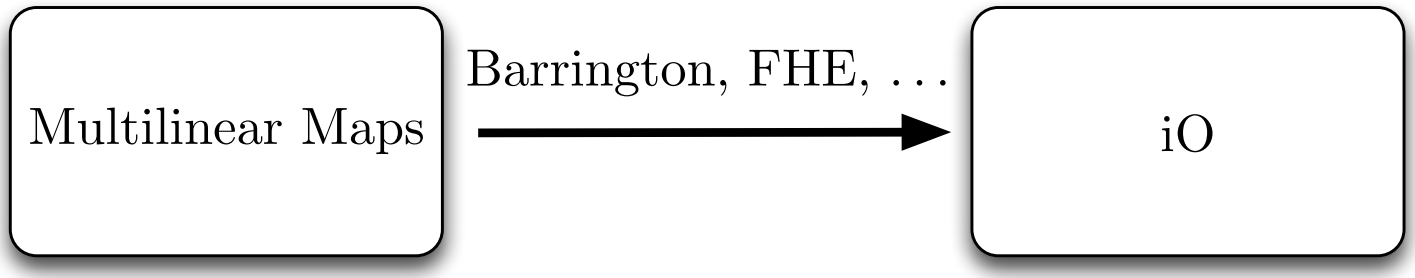
Pooya Farshim

(Joint work with M. Albrecht, D. Hofheinz, E. Larraia and K.G. Paterson)

Queen's University Belfast/École Normale Supérieure

Mathematical Aspects of Computer Science – Foundations of Cryptography

iO Construction



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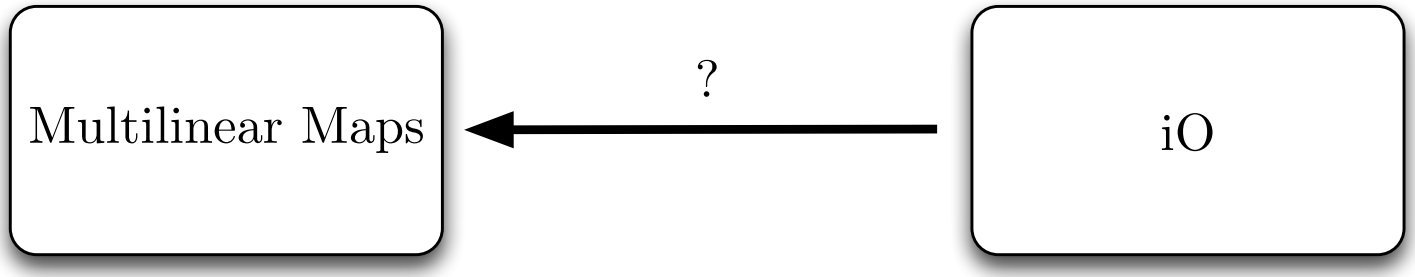

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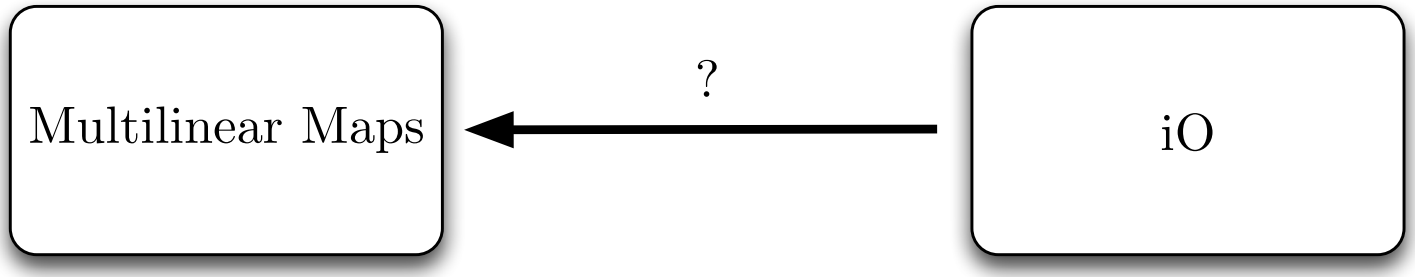

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$$\mathbf{P} = \mathbf{NP} \implies \text{iO Exists} .$$

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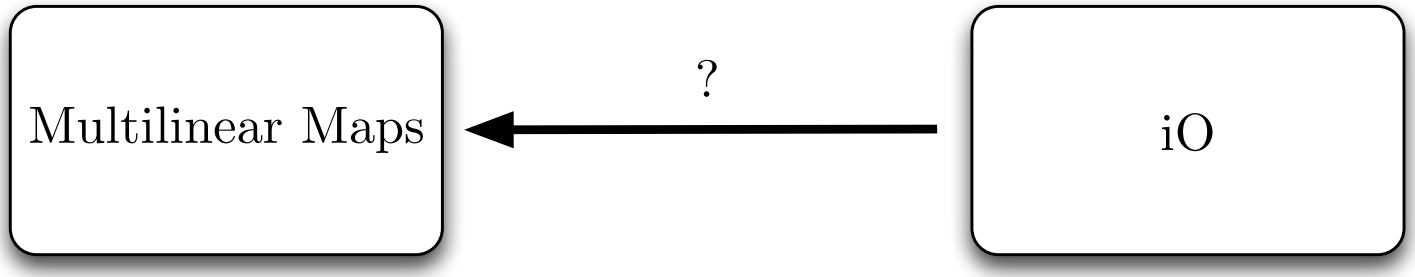

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$$\mathbf{P} = \mathbf{NP} \implies \text{iO Exists} .$$

(Why?) Hence we show

$$\text{iO} + \text{Assumptions} \implies \text{MLM} .$$

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- Theoretical/conceptual:

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- Applications of MLMs
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- Most notably: The current state of MLMs:

  MLM candidates ([GGH13] and [CLT13]) **broken**, but **not** iO!

Multilinear Groups

κ-linear maps:

$$\mathbf{e} : \mathbb{G}_1 \times \cdots \times \mathbb{G}_\kappa \longrightarrow \mathbb{G}_T$$
$$\mathbf{e}(g_1^{a_1}, \ldots, g_\kappa^{a_\kappa}) = g_T^{a_1 \cdots a_\kappa}$$

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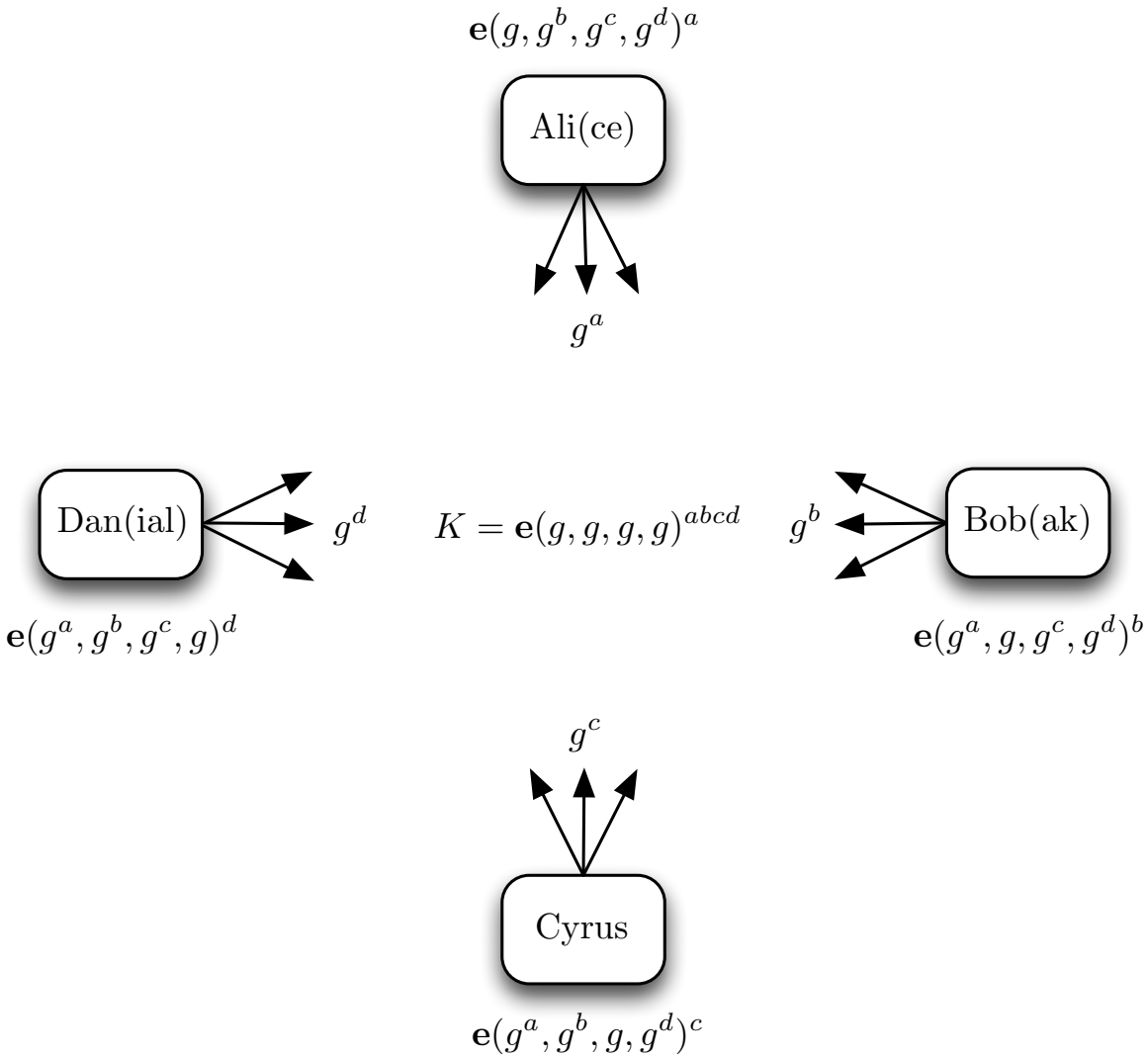
Symmetric setting:

$$\mathbb{G}_i = \mathbb{G}_j = \mathbb{G} \ .$$

Assume for this talk.

NIKE: Non-Interactive Key Exchange



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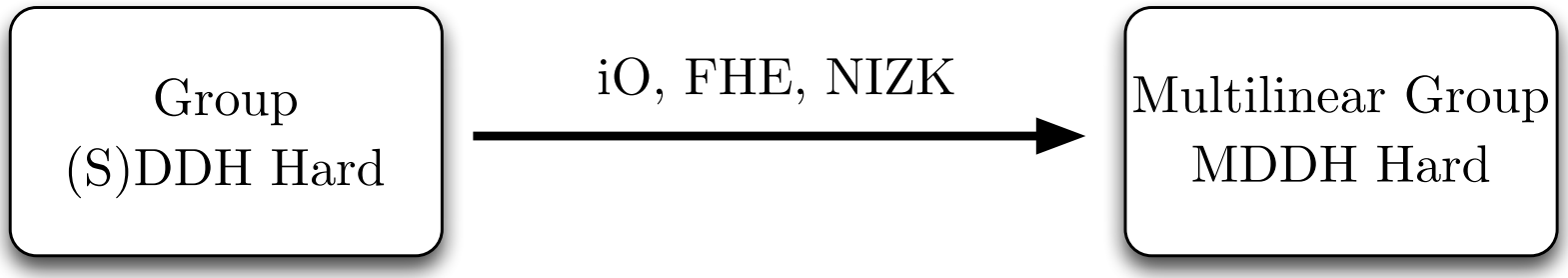

Some Intractable Problems

The DDH Problem:

$$(g, g^{a_1}, g^{a_2}, g^{a_1 a_2}) \overset{c}{\approx} (g, g^{a_1}, g^{a_2}, g^{r})$$

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The (symmetric) κ-MDDH Problem:

$$(g, g^{a_1}, \ldots, g^{a_\kappa}, g^{a_{\kappa+1}}, g_T^{a_1 \cdots a_{\kappa+1}}) \overset{c}{\approx} (g, g^{a_1}, \ldots, g^{a_\kappa}, g^{a_{\kappa+1}}, g_T^{r})$$

Tool 1: FHE

A fully homomorphic public-key encryption scheme

$$(\mathsf{Gen}, \mathsf{Enc}, \mathsf{Eval}, \mathsf{Dec})$$

which is **perfectly** correct wrt. addition in $\mathbb{Z}_p$.

$$\mathsf{Enc}(pk, x_1) \ \text{“+”} \ \mathsf{Enc}(pk, x_2) = \mathsf{Enc}(pk, x_1 + x_2 \pmod p) \ .$$

Perfect Correctness: Needed later to argue for functional equivalence.

Tool 2: Probabilistic iO (piO)

Similar to iO:

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$x \in X$: Computational indistinguishability:

$$C_0(x) \overset{c}{\approx}_{\varepsilon(\lambda)} C_1(x) \quad \text{and} \quad |X(\lambda)| \cdot \varepsilon(\lambda) \in \mathrm{Negl}$$

Construction: Sub-exp. secure iO + sub-exp. secure puncturable PRF [CLTV14].

Tool 3: Dual-Mode NIZK

NIZK with two CRS modes: “binding” BCRS and “hiding” HCRS

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2. Perfect Completeness: BCRS and HCRS
3. Perfect Soundness: BCRS
4. Perfect WI and ZK: HCRS
5. Perfect Extraction: $\text{WExt}(td_{ext}, x, \pi)$ returns w_x under BCRS

Construction: Groth–Sahai Proofs.

The Construction – Intuition

Encode an exponent x as

$$[x] := (g^x, \mathsf{Enc}_1(x), \mathsf{Enc}_2(x), \pi) \ ,$$

analogously the doubly-encrypt-and-prove NY paradigm.

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Group operation:

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Multilinear map: Decrypt $[x_i]$ to get x_i and return

$$\mathbf{e}([x_1], \ldots, [x_\kappa]) := g^{x_1 \cdots x_\kappa} \ .$$

Setup

Group parameters:

$$\sigma,$$

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- TD, a language with hard membership:

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E.g.,

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So $\mathbf{w}_i = (1, w)$ for the **symmetric** setting here.

Group Elements

Encode z as:

$$\left(g^z, \mathsf{Enc}(pk_1, \mathbf{x}_1), \mathsf{Enc}(pk_2, \mathbf{x}_2), \pi\right)$$

where $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{Z}_p^2$ are such that:

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Proof π for

$$\text{Encoding is valid} \quad \textbf{OR} \quad y \in \mathsf{TD}$$

using witness $(\mathbf{x}_1, \mathbf{x}_2, r_1, r_2)$ for the **first** clause.

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A piO of $\mathbf{Add}[sk_1, sk_2, td_{ext}]$ will be made public.

Algorithm $\mathbf{e}[sk_1](h_1, \ldots, h_\kappa)$

- For $i = 1 \ldots \kappa$:
- Parse $([z_i], \mathbf{c}_{i,1}, \mathbf{c}_{i,2}, \pi_i) \leftarrow h_i$
- Check π_i for validity
- Compute $\mathbf{x}_i \leftarrow \mathsf{Dec}(sk_1, \mathbf{c}_{i,1})$
- $z \leftarrow \langle \mathbf{x}_1, \mathbf{w}_1 \rangle \cdots \langle \mathbf{x}_\kappa, \mathbf{w}_\kappa \rangle \pmod{p}$
- Return g^z

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Note:

- Map $\mathbf{e}$ is multilinear.

Algorithm $\mathbf{e}[sk_1](h_1, \ldots, h_\kappa)$

- For $i = 1 \ldots \kappa$:
- Parse $([z_i], \mathbf{c}_{i,1}, \mathbf{c}_{i,2}, \pi_i) \leftarrow h_i$
- Check π_i for validity
- Compute $\mathbf{x}_i \leftarrow \mathsf{Dec}(sk_1, \mathbf{c}_{i,1})$
- $z \leftarrow \langle \mathbf{x}_1, \mathbf{w}_1 \rangle \cdots \langle \mathbf{x}_\kappa, \mathbf{w}_\kappa \rangle \pmod p$
- Return g^z

Note:

- Map $\mathbf{e}$ is multilinear.
- A piO of $\mathbf{e}[sk_1]$ will be made public.

A Proof Gadget: Forgetting (sk_1, sk_2, td_{ext}) for w_y

Goal (1):

$$\mathbf{Add}[sk_1, sk_2, td_{ext}](\cdot,\cdot) \longrightarrow \mathbf{Add}[w_y](\cdot,\cdot)$$

A Proof Gadget: Forgetting (sk_1, sk_2, td_{ext}) for w_y

Goal (1):

$$\mathbf{Add}[sk_1, sk_2, td_{ext}](\cdot,\cdot) \longrightarrow \mathbf{Add}[w_y](\cdot,\cdot)$$

(1) If OK: proof π using (sk_1, sk_2) that $(g^z, \mathbf{c}_1, \mathbf{c}_2)$ is valid.

(2) Else: use td_{ext} to extract $w'_{i,y}$ from π_i for $y \in \mathsf{TD}$. Generate π.

Proof:

G	σ	y	$C_{\mathbf{Add}}$ knows	π- witness	**Remark**

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Proof:

G	σ	y	C_{Add} knows	π- witness	**Remark**
0	binding	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2 or w'_y	

A Proof Gadget: Forgetting (sk_1, sk_2, td_{ext}) for w_y

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$$\mathbf{Add}[sk_1, sk_2, td_{ext}](\cdot,\cdot) \longrightarrow \mathbf{Add}[w_y](\cdot,\cdot)$$

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Proof:

G	σ	y	C_{Add} knows	π- witness	**Remark**
0	binding	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2 or w'_y	
1	binding	$\in$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2 or w'_y	TD indist.

A Proof Gadget: Forgetting (sk_1, sk_2, td_{ext}) for w_y

Goal (1):

$$\mathbf{Add}[sk_1, sk_2, td_{ext}](\cdot,\cdot) \longrightarrow \mathbf{Add}[w_y](\cdot,\cdot)$$

(1) If OK: proof π using (sk_1, sk_2) that $(g^z, \mathbf{c}_1, \mathbf{c}_2)$ is valid.

(2) Else: use td_{ext} to extract $w'_{i,y}$ from π_i for $y \in$ TD. Generate π.

Proof:

G	σ	y	C_{Add} knows	π- witness	**Remark**
0	binding	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2 or w'_y	
1	binding	$\boxed{\in \text{ TD}}$	sk_1, sk_2, td_{ext}	sk_1, sk_2 or w'_y	TD indist.
2	binding	$\in$ TD	$\boxed{sk_1, sk_2, w_y}$	sk_1, sk_2 or w_y (2)	piO (unique w_y + prefect soundness)

A Proof Gadget: Forgetting (sk_1, sk_2, td_{ext}) for w_y

Goal (1):

$$\mathbf{Add}[sk_1, sk_2, td_{ext}](\cdot,\cdot) \longrightarrow \mathbf{Add}[w_y](\cdot,\cdot)$$

(1) If OK: proof π using (sk_1, sk_2) that $(g^z, \mathbf{c}_1, \mathbf{c}_2)$ is valid.

(2) Else: use td_{ext} to extract $w'_{i,y}$ from π_i for $y \in$ TD. Generate π.

Proof:

G	σ	y	C_{Add} knows	π-witness	**Remark**
0	binding	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2 or w'_y	
1	binding	$\boxed{\in \text{ TD}}$	sk_1, sk_2, td_{ext}	sk_1, sk_2 or w'_y	TD indist.
2	binding	$\in$ TD	$\boxed{sk_1, sk_2, w_y}$	sk_1, sk_2 or w_y (2)	piO (unique w_y + prefect soundness)
3	$\boxed{\text{hiding}}$	$\in$ TD	sk_1, sk_2, w_y	sk_1, sk_2 or w_y (2)	CRS indist.

A Proof Gadget: Forgetting (sk_1, sk_2, td_{ext}) for w_y

Goal (1):

$$\mathbf{Add}[sk_1, sk_2, td_{ext}](\cdot,\cdot) \longrightarrow \mathbf{Add}[w_y](\cdot,\cdot)$$

(1) If OK: proof π using (sk_1, sk_2) that $(g^z, \mathbf{c}_1, \mathbf{c}_2)$ is valid.

(2) Else: use td_{ext} to extract $w'_{i,y}$ from π_i for $y \in$ TD. Generate π.

Proof:

G	σ	y	C_{Add} knows	π- witness	**Remark**
0	binding	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2 or w'_y	
1	binding	$\boxed{\in \text{ TD}}$	sk_1, sk_2, td_{ext}	sk_1, sk_2 or w'_y	TD indist.
2	binding	$\in$ TD	$\boxed{sk_1, sk_2, w_y}$	sk_1, sk_2 or w_y (2)	piO (unique w_y + prefect soundness)
3	$\boxed{\text{hiding}}$	$\in$ TD	sk_1, sk_2, w_y	sk_1, sk_2 or w_y (2)	CRS indist.
4	hiding	$\in$ TD	$\boxed{w_y}$	$\boxed{w_y \text{ (1)} + \text{(2)}}$	piO + perfect WI

Switching Encodings

Goal (2):

$$\big(\mathsf{Enc}(pk_1, \mathbf{x}_0), \mathsf{Enc}(pk_2, \mathbf{y}_0)\big) \longrightarrow \big(\mathsf{Enc}(pk_1, \mathbf{x}_1), \mathsf{Enc}(pk_2, \mathbf{y}_1)\big)$$

Switching Encodings

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Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**

Switching Encodings

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$$\big(\mathsf{Enc}(pk_1, \mathbf{x}_0), \mathsf{Enc}(pk_2, \mathbf{y}_0)\big) \longrightarrow \big(\mathsf{Enc}(pk_1, \mathbf{x}_1), \mathsf{Enc}(pk_2, \mathbf{y}_1)\big)$$

Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	

Switching Encodings

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Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	

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Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget

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Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	

Switching Encodings

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Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	IND-CPA wrt. pk_2

Switching Encodings

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Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	IND-CPA wrt. pk_2
3	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	

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G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	IND-CPA wrt. pk_2
3	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget

Switching Encodings

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Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	IND-CPA wrt. pk_2
3	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
4	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	

Switching Encodings

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Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	IND-CPA wrt. pk_2
3	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
4	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	iO (soundness)

Switching Encodings

Goal (2):

$$\big(\mathsf{Enc}(pk_1, \mathbf{x}_0), \mathsf{Enc}(pk_2, \mathbf{y}_0)\big) \longrightarrow \big(\mathsf{Enc}(pk_1, \mathbf{x}_1), \mathsf{Enc}(pk_2, \mathbf{y}_1)\big)$$

Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	**hide**	**$\in$ TD**	**w_y**	**w_y**	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	**$\mathbf{y}_1$**	IND-CPA wrt. pk_2
3	**bind**	**$\notin$ TD**	**sk_1, sk_2, td_{ext}**	**sk_1, sk_2**	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
4	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	**sk_2**	$\mathbf{x}_0$	$\mathbf{y}_1$	iO (soundness)
5	**hide**	**$\in$ TD**	**w_y**	**w_y**	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	

Switching Encodings

Goal (2):

$$\big(\mathsf{Enc}(pk_1, \mathbf{x}_0), \mathsf{Enc}(pk_2, \mathbf{y}_0)\big) \longrightarrow \big(\mathsf{Enc}(pk_1, \mathbf{x}_1), \mathsf{Enc}(pk_2, \mathbf{y}_1)\big)$$

Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	IND-CPA wrt. pk_2
3	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
4	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	iO (soundness)
5	hide	$\in$ TD	w_y	w_y	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget

Switching Encodings

Goal (2):

$$\big(\mathsf{Enc}(pk_1, \mathbf{x}_0), \mathsf{Enc}(pk_2, \mathbf{y}_0)\big) \longrightarrow \big(\mathsf{Enc}(pk_1, \mathbf{x}_1), \mathsf{Enc}(pk_2, \mathbf{y}_1)\big)$$

Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	IND-CPA wrt. pk_2
3	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
4	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	iO (soundness)
5	hide	$\in$ TD	w_y	w_y	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
6	hide	$\in$ TD	w_y	w_y	sk_2	$\mathbf{x}_1$	$\mathbf{y}_1$	

Switching Encodings

Goal (2):

$$\big(\mathsf{Enc}(pk_1, \mathbf{x}_0), \mathsf{Enc}(pk_2, \mathbf{y}_0)\big) \longrightarrow \big(\mathsf{Enc}(pk_1, \mathbf{x}_1), \mathsf{Enc}(pk_2, \mathbf{y}_1)\big)$$

Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	IND-CPA wrt. pk_2
3	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
4	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	iO (soundness)
5	hide	$\in$ TD	w_y	w_y	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
6	hide	$\in$ TD	w_y	w_y	sk_2	$\mathbf{x}_1$	$\mathbf{y}_1$	IND-CPA wrt. pk_1

Switching Encodings

Goal (2):

$$\big(\mathsf{Enc}(pk_1, \mathbf{x}_0), \mathsf{Enc}(pk_2, \mathbf{y}_0)\big) \longrightarrow \big(\mathsf{Enc}(pk_1, \mathbf{x}_1), \mathsf{Enc}(pk_2, \mathbf{y}_1)\big)$$

Proof:

G	σ	y	C_{Add} knows	π-witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	**hide**	**$\in$ TD**	**w_y**	**w_y**	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	**$\mathbf{y}_1$**	IND-CPA wrt. pk_2
3	**bind**	**$\notin$ TD**	**sk_1, sk_2, td_{ext}**	**sk_1, sk_2**	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
4	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	**sk_2**	$\mathbf{x}_0$	$\mathbf{y}_1$	iO (soundness)
5	**hide**	**$\in$ TD**	**w_y**	**w_y**	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
6	hide	$\in$ TD	w_y	w_y	sk_2	**$\mathbf{x}_1$**	$\mathbf{y}_1$	IND-CPA wrt. pk_1
7	**bind**	**$\notin$ TD**	**sk_1, sk_2, td_{ext}**	**sk_1, sk_2**	sk_2	$\mathbf{x}_1$	$\mathbf{y}_1$	

Switching Encodings

Goal (2):

$$\big(\mathsf{Enc}(pk_1, \mathbf{x}_0), \mathsf{Enc}(pk_2, \mathbf{y}_0)\big) \longrightarrow \big(\mathsf{Enc}(pk_1, \mathbf{x}_1), \mathsf{Enc}(pk_2, \mathbf{y}_1)\big)$$

Proof:

G	σ	y	C_{Add} knows	π-witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	IND-CPA wrt. pk_2
3	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
4	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	iO (soundness)
5	hide	$\in$ TD	w_y	w_y	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
6	hide	$\in$ TD	w_y	w_y	sk_2	$\mathbf{x}_1$	$\mathbf{y}_1$	IND-CPA wrt. pk_1
7	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_2	$\mathbf{x}_1$	$\mathbf{y}_1$	proof gadget

Switching Encodings

Goal (2):

$$\big(\mathsf{Enc}(pk_1, \mathbf{x}_0), \mathsf{Enc}(pk_2, \mathbf{y}_0)\big) \longrightarrow \big(\mathsf{Enc}(pk_1, \mathbf{x}_1), \mathsf{Enc}(pk_2, \mathbf{y}_1)\big)$$

Proof:

G	σ	y	C_{Add} knows	π- witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	**hide**	**$\in$ TD**	**w_y**	**w_y**	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	**$\mathbf{y}_1$**	IND-CPA wrt. pk_2
3	**bind**	**$\notin$ TD**	**sk_1, sk_2, td_{ext}**	**sk_1, sk_2**	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
4	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	**sk_2**	$\mathbf{x}_0$	$\mathbf{y}_1$	iO (soundness)
5	**hide**	**$\in$ TD**	**w_y**	**w_y**	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
6	hide	$\in$ TD	w_y	w_y	sk_2	**$\mathbf{x}_1$**	$\mathbf{y}_1$	IND-CPA wrt. pk_1
7	**bind**	**$\notin$ TD**	**sk_1, sk_2, td_{ext}**	**sk_1, sk_2**	sk_2	$\mathbf{x}_1$	$\mathbf{y}_1$	proof gadget
8	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	**sk_1**	$\mathbf{x}_1$	$\mathbf{y}_1$	

Switching Encodings

Goal (2):

$$\big(\mathsf{Enc}(pk_1, \mathbf{x}_0), \mathsf{Enc}(pk_2, \mathbf{y}_0)\big) \longrightarrow \big(\mathsf{Enc}(pk_1, \mathbf{x}_1), \mathsf{Enc}(pk_2, \mathbf{y}_1)\big)$$

Proof:

G	σ	y	C_{Add} knows	π-witness	C_{Map} knows	$\mathbf{c}_1$ contains	$\mathbf{c}_2$ contains	**Remark**
0	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	
1	**hide**	**$\in$ TD**	**w_y**	**w_y**	sk_1	$\mathbf{x}_0$	$\mathbf{y}_0$	proof gadget
2	hide	$\in$ TD	w_y	w_y	sk_1	$\mathbf{x}_0$	**$\mathbf{y}_1$**	IND-CPA wrt. pk_2
3	**bind**	**$\notin$ TD**	**sk_1, sk_2, td_{ext}**	**sk_1, sk_2**	sk_1	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
4	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	**sk_2**	$\mathbf{x}_0$	$\mathbf{y}_1$	iO (soundness)
5	**hide**	**$\in$ TD**	**w_y**	**w_y**	sk_2	$\mathbf{x}_0$	$\mathbf{y}_1$	proof gadget
6	hide	$\in$ TD	w_y	w_y	sk_2	**$\mathbf{x}_1$**	$\mathbf{y}_1$	IND-CPA wrt. pk_1
7	**bind**	**$\notin$ TD**	**sk_1, sk_2, td_{ext}**	**sk_1, sk_2**	sk_2	$\mathbf{x}_1$	$\mathbf{y}_1$	proof gadget
8	bind	$\notin$ TD	sk_1, sk_2, td_{ext}	sk_1, sk_2	**sk_1**	$\mathbf{x}_1$	$\mathbf{y}_1$	iO (soundness)

Computing $\mathbf{e}$ with Implicit $[\mathbf{W}]$

Recall

$$\mathbf{e}([z_1], \ldots, [z_\kappa]) = g^{\langle \mathbf{x}_1, \mathbf{w}_1 \rangle \cdots \langle \mathbf{x}_\kappa, \mathbf{w}_\kappa \rangle}$$

This can be computed using $(\mathbf{x}_1, \ldots, \mathbf{x}_\kappa)$ and **implicit** values

$$g, g^{w}, g^{w^2}, \ldots, g^{w^\kappa} \ .$$

How? Recall $\mathbf{w}_i = (1, w)$

$$\prod_{j=1}^{\kappa} \langle \mathbf{x}_i, \mathbf{w}_i \rangle = \prod_{j=1}^{\kappa} (x_{i,0} + x_{i,1} \cdot w) = \sum_{j=1}^{\kappa} \alpha_j w^j$$

And all we need are the coefficients α_j, which we can compute iteratively.

Main Proof: Hardness of Symmetric κ-MDDH

Final Goal:

$$g^{a_1 \cdots a_{\kappa+1}} \overset{c}{\approx} g^r$$

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Proof:

$[a_1]$	$[a_2]$	$\cdots$	$[a_\kappa]$	$[a_{\kappa+1}]$	MDDH exponent	C_{Map}	**Remark**

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Proof:

$[a_1]$	$[a_2]$	$\cdots$	$[a_\kappa]$	$[a_{\kappa+1}]$	MDDH exponent	C_{Map}	**Remark**
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)

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$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	

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$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
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$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	

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$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch

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$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
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$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

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$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

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$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	

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$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch

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$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	

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$[a_1]$	$[a_2]$	$\cdots$	$[a_\kappa]$	$[a_{\kappa+1}]$	MDDH exponent	C_{Map}	**Remark**
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	Renaming

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$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \sum_{j=1}^{\kappa} \alpha_j w^j$	$\boxed{[w^j] = g^{w^j}}$	

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$[a_1]$	$[a_2]$	$\cdots$	$[a_\kappa]$	$[a_{\kappa+1}]$	MDDH exponent	C_{Map}	**Remark**
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \sum_{j=1}^{\kappa} \alpha_j w^j$	$\boxed{[w^j] = g^{w^j}}$	iO

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$[a_1]$	$[a_2]$	$\cdots$	$[a_\kappa]$	$[a_{\kappa+1}]$	MDDH exponent	C_{Map}	**Remark**
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \sum_{j=1}^{\kappa} \alpha_j w^j$	$\boxed{[w^j] = g^{w^j}}$	iO
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1}(\boxed{\mathsf{r}} + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$\boxed{g^{w^\kappa} \to g^r}$	

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$[a_1]$	$[a_2]$	$\cdots$	$[a_\kappa]$	$[a_{\kappa+1}]$	MDDH exponent	C_{Map}	**Remark**
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \sum_{j=1}^{\kappa} \alpha_j w^j$	$\boxed{[w^j] = g^{w^j}}$	iO
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1}(\boxed{\mathsf{r}} + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$\boxed{g^{w^\kappa} \to g^r}$	$(\kappa - 1)$-SDDH

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$[a_1]$	$[a_2]$	$\cdots$	$[a_\kappa]$	$[a_{\kappa+1}]$	MDDH exponent	C_{Map}	**Remark**
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \sum_{j=1}^{\kappa} \alpha_j w^j$	$\boxed{[w^j] = g^{w^j}}$	iO
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1}(\boxed{r} + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$\boxed{g^{w^\kappa} \to g^r}$	$(\kappa - 1)$-SDDH
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$\boxed{(a_{\kappa+1} - w, 0)}$	$a_{\kappa+1}(r + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$g^{w^\kappa} \to g^r$	

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$$g^{a_1 \cdots a_{\kappa+1}} \overset{c}{\approx} g^r$$

Proof:

$[a_1]$	$[a_2]$	$\cdots$	$[a_\kappa]$	$[a_{\kappa+1}]$	MDDH exponent	C_{Map}	**Remark**
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \sum_{j=1}^{\kappa} \alpha_j w^j$	$\boxed{[w^j] = g^{w^j}}$	iO
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1}(\boxed{\mathsf{r}} + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$\boxed{g^{w^\kappa} \to g^r}$	$(\kappa - 1)$-SDDH
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$\boxed{(a_{\kappa+1} - w, 0)}$	$a_{\kappa+1}(r + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$g^{w^\kappa} \to g^r$	Encoding switch

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$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \sum_{j=1}^{\kappa} \alpha_j w^j$	$\boxed{[w^j] = g^{w^j}}$	iO
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1}(\boxed{\mathsf{r}} + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$\boxed{g^{w^\kappa} \to g^r}$	$(\kappa - 1)$-SDDH
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$\boxed{(a_{\kappa+1} - w, 0)}$	$a_{\kappa+1}(r + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$g^{w^\kappa} \to g^r$	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$\sum_{j=1}^{\kappa} \beta_j w^j$	$g^{w^\kappa} \to g^r$	

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$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \sum_{j=1}^{\kappa} \alpha_j w^j$	$\boxed{[w^j] = g^{w^j}}$	iO
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1}(\boxed{\mathsf{r}} + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$\boxed{g^{w^\kappa} \to g^r}$	$(\kappa - 1)$-SDDH
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$\boxed{(a_{\kappa+1} - w, 0)}$	$a_{\kappa+1}(r + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$g^{w^\kappa} \to g^r$	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$\sum_{j=1}^{\kappa} \beta_j w^j$	$g^{w^\kappa} \to g^r$	Renaming

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$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \sum_{j=1}^{\kappa} \alpha_j w^j$	$\boxed{[w^j] = g^{w^j}}$	iO
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1}(\boxed{\mathsf{r}} + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$\boxed{g^{w^\kappa} \to g^r}$	$(\kappa - 1)$-SDDH
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$\boxed{(a_{\kappa+1} - w, 0)}$	$a_{\kappa+1}(r + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$g^{w^\kappa} \to g^r$	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$\sum_{j=1}^{\kappa} \beta_j w^j$	$g^{w^\kappa} \to g^r$	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$\boxed{s + \sum_{j=1}^{\kappa-1} \beta_j w^j}$	$g^{w^\kappa} \to g^r$	

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$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	MDDH ($b = 1$)
$\boxed{(a_1 - w, 0)}$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1 - w, 0)$	$\boxed{(a_2 - w, 0)}$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \prod_{i=1}^{\kappa} (a_i + w)$	w	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1} \sum_{j=1}^{\kappa} \alpha_j w^j$	$\boxed{[w^j] = g^{w^j}}$	iO
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1}(\boxed{\mathsf{r}} + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$\boxed{g^{w^\kappa} \to g^r}$	$(\kappa - 1)$-SDDH
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$\boxed{(a_{\kappa+1} - w, 0)}$	$a_{\kappa+1}(r + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$g^{w^\kappa} \to g^r$	Encoding switch
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$\sum_{j=1}^{\kappa} \beta_j w^j$	$g^{w^\kappa} \to g^r$	Renaming
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$\boxed{s + \sum_{j=1}^{\kappa-1} \beta_j w^j}$	$g^{w^\kappa} \to g^r$	$(\kappa - 1)$-SDDH

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$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
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$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$a_{\kappa+1}(\boxed{\mathsf{r}} + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$\boxed{g^{w^\kappa} \to g^r}$	$(\kappa - 1)$-SDDH
$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$\boxed{(a_{\kappa+1} - w, 0)}$	$a_{\kappa+1}(r + \sum_{j=1}^{\kappa-1} \alpha_j w^j)$	$g^{w^\kappa} \to g^r$	Encoding switch
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$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	$\boxed{s + \sum_{j=1}^{\kappa-1} \beta_j w^j}$	$g^{w^\kappa} \to g^r$	$(\kappa - 1)$-SDDH
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$(a_1 - w, 0)$	$(a_2 - w, 0)$	$\cdots$	$\boxed{(a_\kappa - w, 0)}$	$(a_{\kappa+1}, 0)$	$a_1 \cdots a_{\kappa+1}$	w	Encoding switch
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$(a_1, 0)$	$(a_2, 0)$	$\cdots$	$(a_\kappa, 0)$	$(a_{\kappa+1}, 0)$	s	$g^{w^\kappa} \to g^r$	Renaming

Extensions and Open Problems

See paper (ePrint 2015/780) for:

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Thank you.