اورکل‌های تصادفی و مبهم‌سازی کد

پویا فرشیم

(کار مشترک با کریستینا بروژکا و آرنو میتل‌باخ)

دانشگاه کوئینز بلفاست – اکول نرمال سوپریور

جنبه‌های ریاضی علوم کامپیوتری – مبانی رمز

Random Oracles and Obfuscation

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Queen's University Belfast/École Normale Supérieure

Mathematical Aspects of Computer Science – Foundations of Cryptography

Random Oracles

- Random oracles (ROs) model ideal hash functions [BR93].
  In the RO model:

  All parties have oracle access to a
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- ROs enable the security proofs of a wide range of practical and strongly secure cryptosystems: encryption & signature schemes, key exchange, disk encryption, ...
- Standard-model counterparts are often less secure and/or less efficient.

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- Reliance on ROs, although practical, is somewhat debatable:

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Scheme $\mathsf{Enc}^{O}(K, M)$:

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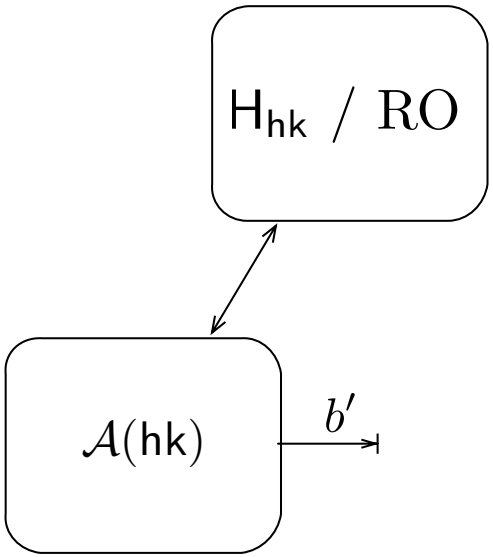
  3. Else return a normal/good encryption of M.
- Lack of a **definition** formalizing "RO-like" behavior.

(Very) Naïve Attempt at Modeling ROs

Call a hash function "IND-RO" if:



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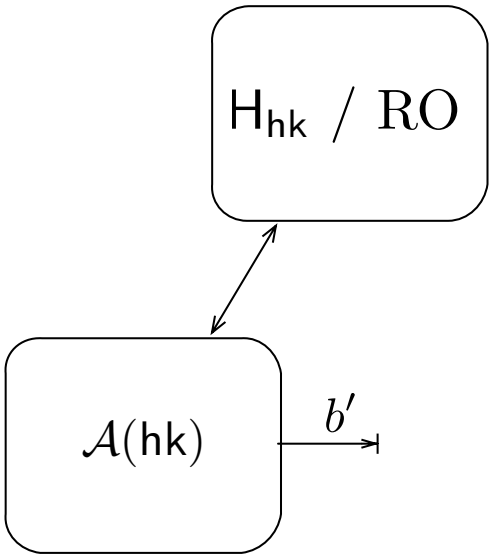

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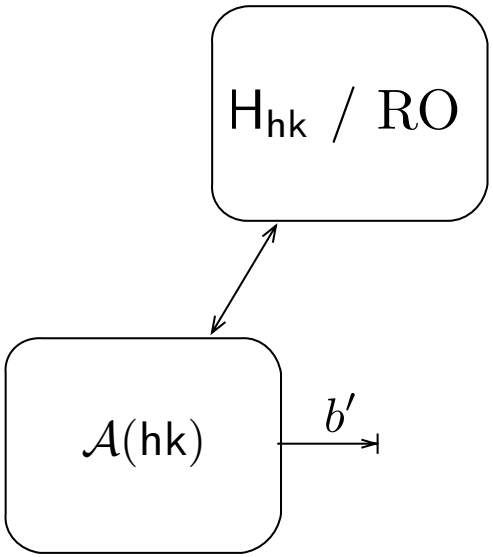

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Clearly uninstantiable:

$\mathcal{A}(\mathsf{hk})$: compute $\mathsf{H}_{\mathsf{hk}}(0)$ and compare to the oracle's reply.

But observe:

The adversary knows a full input, namely $(\mathsf{hk}, 0)$.

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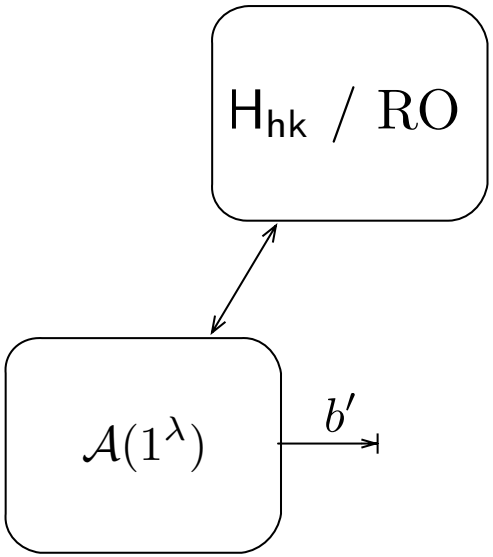

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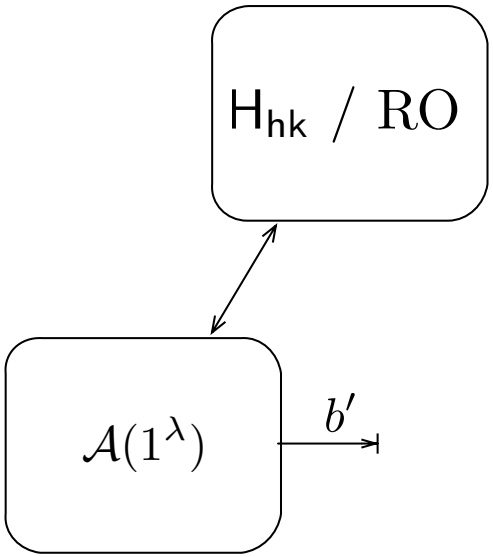

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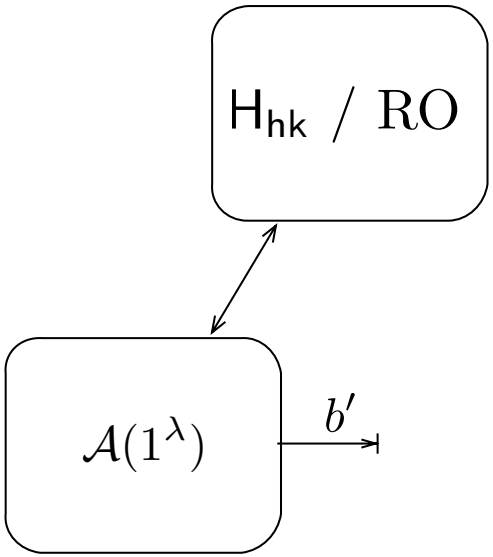

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First idea:

Split $\mathcal{A}$: one part gets hk and the other gets oracle access.

Modeling ROs via Split Adversaries

Call the two components of $\mathcal{A}$ the **source** $\mathcal{S}$ and the **distinguisher** $\mathcal{D}$:

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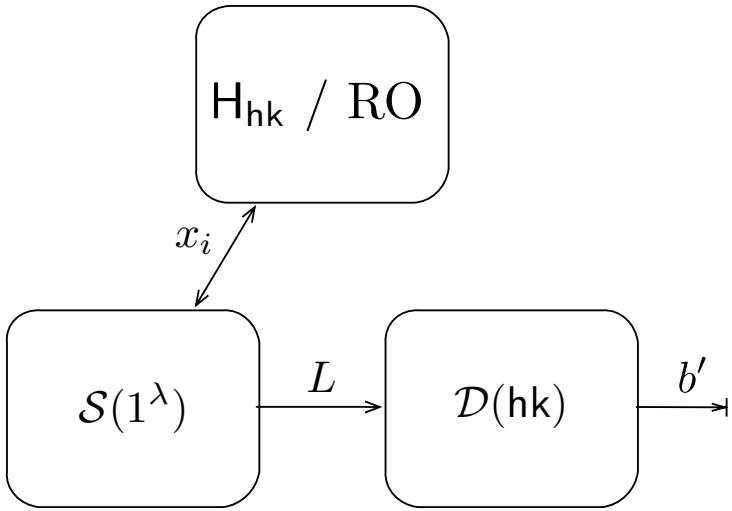

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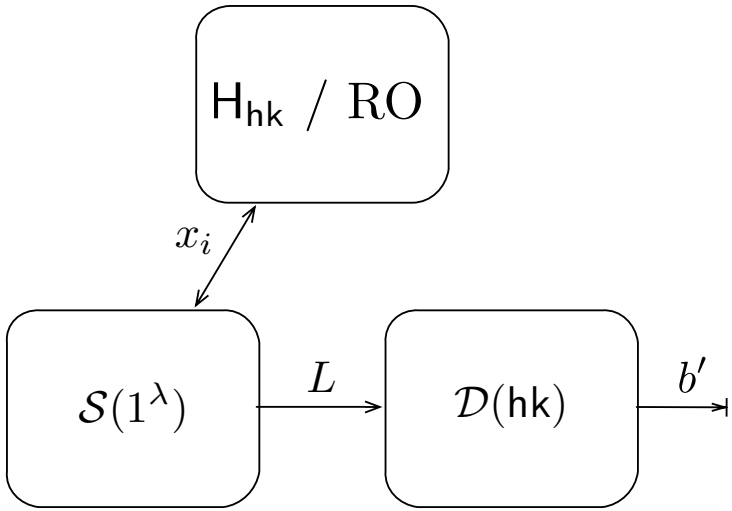

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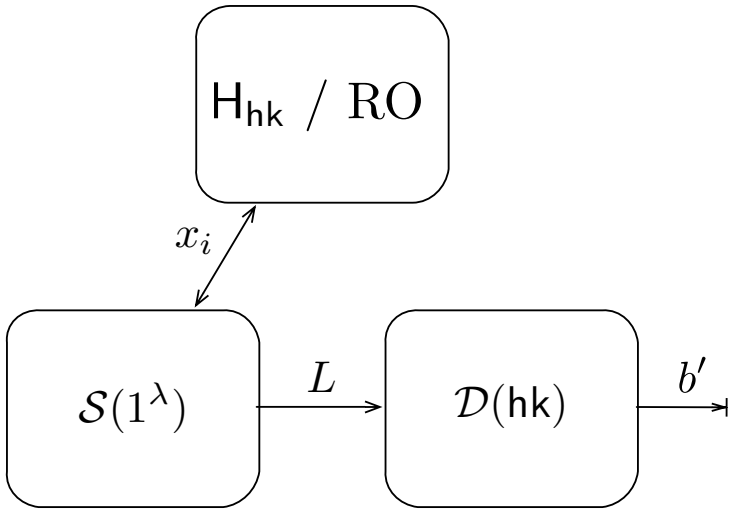

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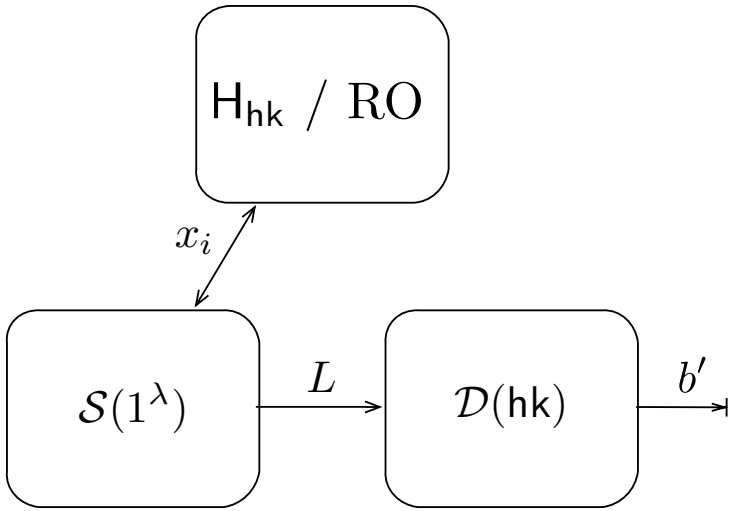

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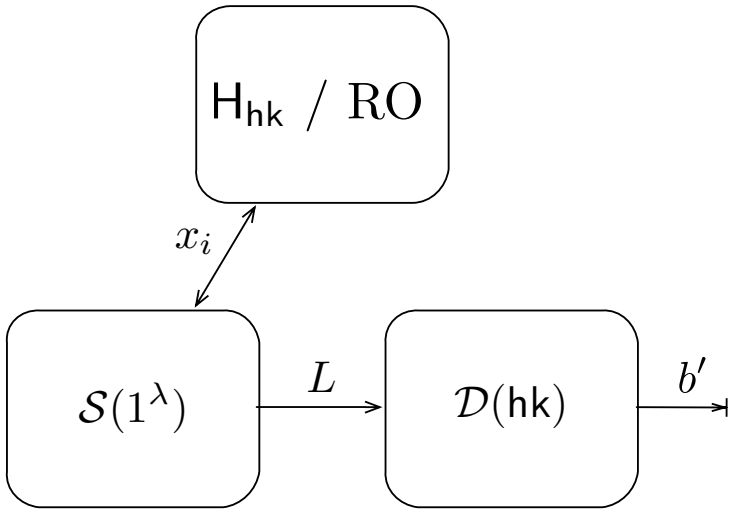
Second idea:

Restrict L: it must not leak any of $\mathcal{S}$'s queries.

Universal Computational Extractors (UCEs) [BHK13a]



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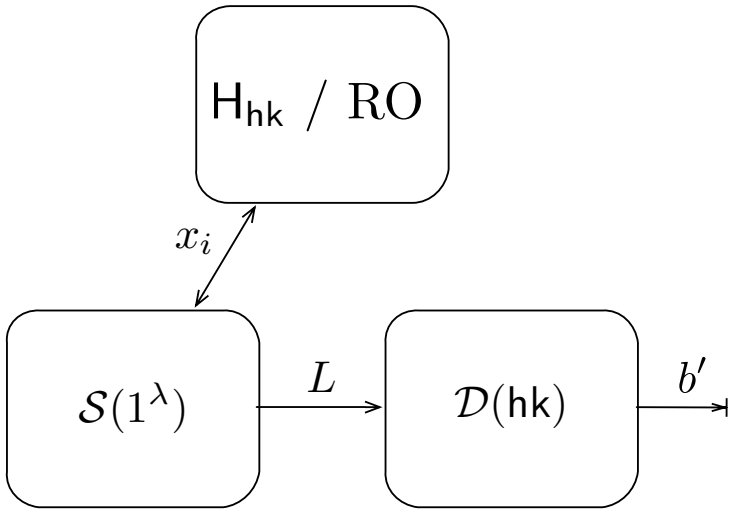

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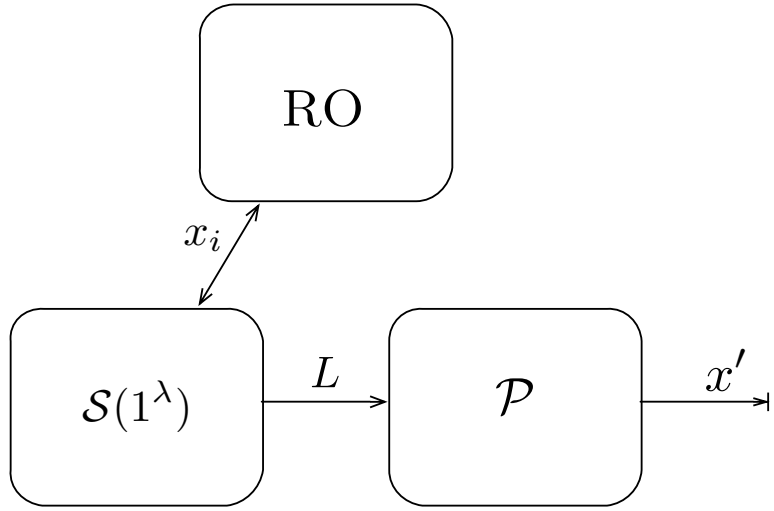

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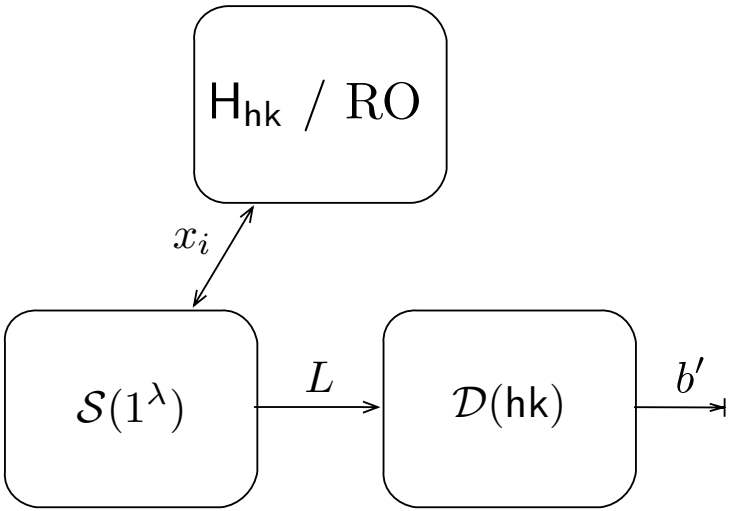

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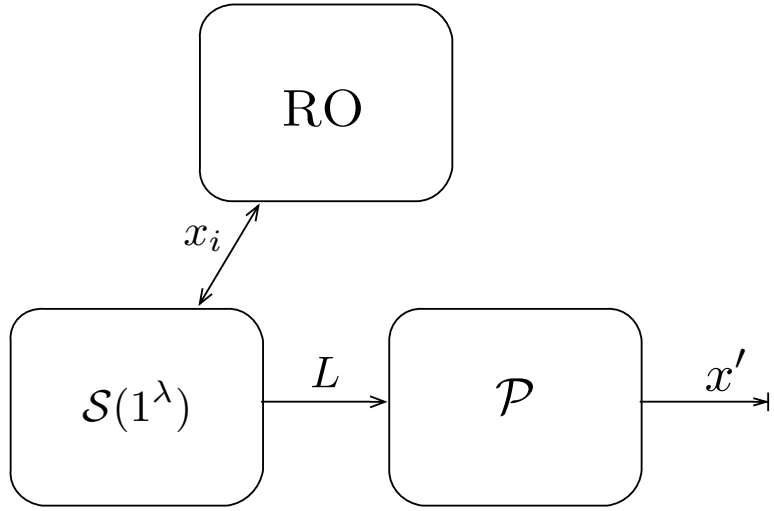

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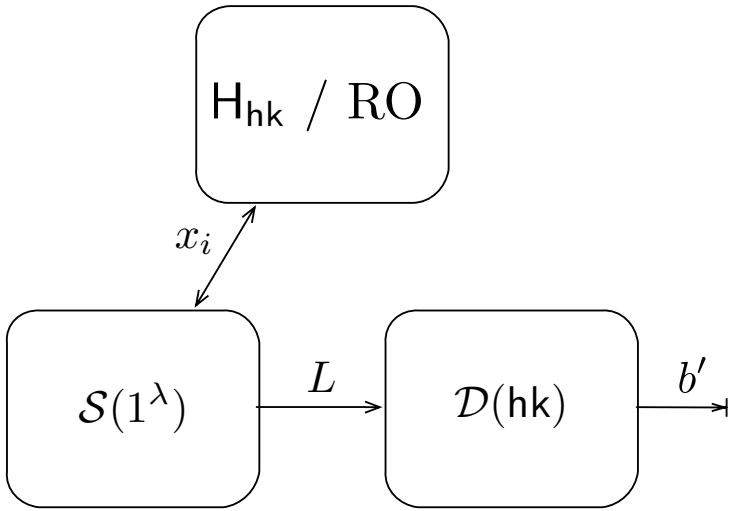

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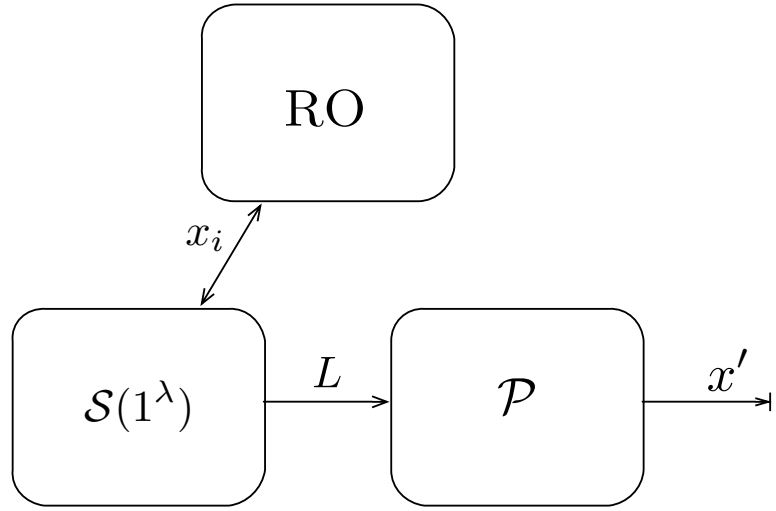

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H is **UCE1 secure** iff: $\mathbf{Adv}^{\mathrm{uce}}_{\mathsf{H},\mathcal{S},\mathcal{D}}(\lambda) \in \mathrm{Negl}$ for any **unpredictable** $\mathcal{S}$.

Applications of UCE [BHK13a]

UCE-secure hash functions can instantiate the RO in

- Deterministic PKEs
- RKA and KDM security
- Point-function obfuscation
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- Proofs of storage
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Bottom line:

UCEs model many RO-like properties.

Is UCE Security Instantiable?

Suppose we could "strongly" obfuscate H:

- $\mathcal{S}$: Choose a random x. Query x to get y. Leak as L the values

$$y, \quad \mathsf{Obf}(\mathsf{H}(\cdot, x)) \ .$$

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Perhaps a different circuit and/or obfuscator might help?

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Let C_0 and C_1 be two **functionally equivalent** circuits:

$$\forall x : C_0(x) = C_1(x) \ .$$

iO **security**: cannot efficiently distinguish obfuscations of such circuits:

$$\mathsf{iO}(C_0) \stackrel{c}{\approx} \mathsf{iO}(C_1) \ .$$

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Can we use iO to attack UCEs?

The iO Attack

$\mathcal{S}$: Leak an indistinguishably obfuscation of the Boolean circuit

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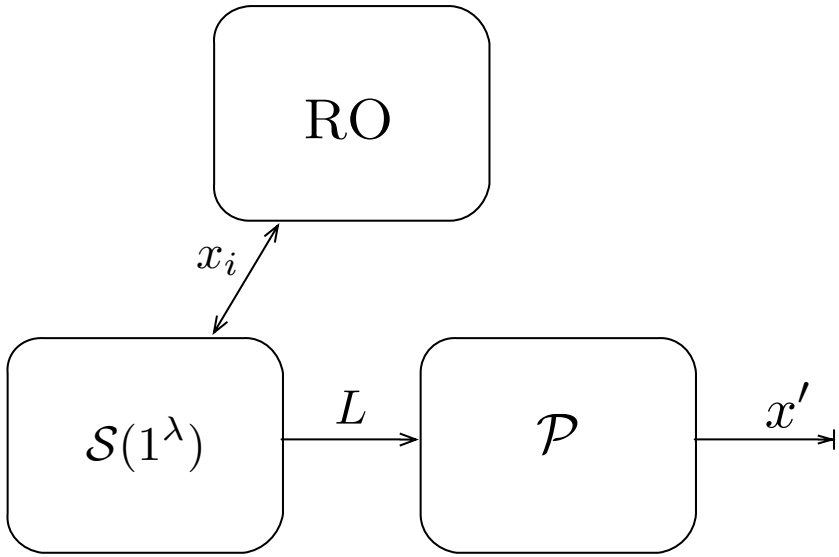

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$$\mathsf{iO}(\mathsf{H}(\cdot, x) \stackrel{?}{=} y) \quad \text{leaks no more than} \quad \mathsf{iO}(\mathsf{Zero}) \ ,$$

and the latter is independent of x.

Dropping $|\mathsf{KSp}| \ll |\mathsf{Rng}|$

Consider instead iO-ing the circuit

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Theorem

Under the existence of iO, UCE security is uninstantiable.

Remark: Does not contradict UCE instantiability in the RO model.

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Salvaging UCE: Statistical Unpredictability

iO is only **computationally** secure. Restrict the source class by

Allowing the **predictors** to run in **unbounded** time.

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Remark for researchers working on negative results:

Example of an impossibility result that is
used positively to define a security model.

Statistical UCE and Applications

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Does **not** hold for D-PKE, MLE, OAEP, and hard-core functions:

Reductions to UCE rely on L which only **computationally** hide x.

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Following our attack [BHK13c] introduced two new **computational** UCE notions:

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Utilizing **randomized encodings** [AIK06] we show:

Theorem

Under iO and PRG bounded parallel UCE security is uninstantiable.

Deterministic PKEs

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Security: IND wrt **high-entropy and** pk-**independent** messages:

$$\mathsf{DetEnc}(\mathsf{pk}, m_0) \overset{c}{\approx} \mathsf{DetEnc}(\mathsf{pk}, m_1) \ ,$$

where

$$(m_0, m_1) \leftarrow\!\!\$\ \mathcal{M}(1^\lambda)$$

and both m_0 and m_1 are required to have high min-entropy.

Encrypt-with-Hash (EwH)

EwH converts a normal PKE to a D-PKE:

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Our attacks say: this theorem is vacuous. This raises the questions:

Can EwH instantiated under some other UCE-type assumption?
Or is EwH uninstantiable to start with?

Uninstantiability

Steps to show the uninstantiability of EwH:

- Start with an CPA scheme PKE.
- **Tweak** this scheme to a new scheme PKE^*.
- Prove that PKE^* is still CPA secure.
- Show that using PKE^* in EwH results in an **insecure** D-PKE for **any** hash function used to instantiate the random oracle.

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So what's the tweak?

Pushing the Attack down to EwH

Recall the circuit used to attack UCEs:

$$\mathsf{P}[x, y]\,(\cdot) : \text{if } \mathsf{H}(\cdot, x) = y \text{ return } 1 \text{ else return } 0\ .$$

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Now let's look at a **universal** variant of this circuit:

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The tweaked scheme PKE^* is as follows:

- Gen^*: identical to that of the base scheme PKE.
- Enc^*: compute c as in PKE, but also output iO of $\mathsf{P}'[\mathsf{pk}, m, r]$:

  Enc^* is playing the role of a **UCE source**.
- Dec^*: ignore the iO component and PKE-decrypt.

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Almost:

- **Attack**: run the circuit on the description of the hash function used in instantiation to recover m in the clear.
- **CPA security of** PKE^*: try to argue as before that

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Cannot apply the union-bound argument directly:

There are potentially many (possibly infinity) more hash functions than hash digests.

Tweaking the Circuit

Consider a PKE that uses a **sparse** set of coins:

Coins r are first mapped to $\mathsf{PRG}(r)$.
They are then used within the scheme.

Adapt the circuit accordingly:

$$\mathsf{P}[\mathsf{pk}, m, \mathsf{PRG}(r)]\left(\mathsf{H}(\mathsf{hk}, \cdot)\right) : \text{if } \mathsf{PRG}(\mathsf{H}(\mathsf{hk}, \mathsf{pk}\|m)) = \mathsf{PRG}(r) \text{ return } m$$

$$\text{else return } 0$$

How Do We Argue Now?

Two game hops:

1. **PRG hop:** The **description** of $\mathsf{P}[\mathsf{pk}, m, \mathsf{PRG}(r)]$ is ind. from $\mathsf{P}[\mathsf{pk}, m, s]$ for a truly random s.
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Structurally,

$$\mathsf{P}_{\mathsf{PRG}(r)} \overset{c}{\approx} \mathsf{P}_s \equiv \mathsf{Z}$$

$$i\mathcal{O}\left(\mathsf{P}_{\mathsf{PRG}(r)}\right) \overset{c}{\approx} i\mathcal{O}\left(\mathsf{P}_s\right) \overset{c}{\approx} i\mathcal{O}\left(\mathsf{Z}\right)$$

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P takes as input a hash-function description:

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Assumptions:

- [GGHRSW14] constructs iO for circuits.
- iO for TMs can be based on (pubic-coin) **differing-inputs** obfuscation and a few other high-tech gadgets [IPS15,ABGSZ13,BCP13].

Flexibility of the Technique

- Generalizes to a wider class of "admissible" transforms.
- Applies to the classical **Fujisaki–Okamoto** transform [FO98] for converting CPA to CCA security.
- Can be adapted to ROM transforms for **KDM** security, **message-locked** encryption, and **hedged** PKEs.
- Schemes specifically designed to foil our simple iO attack fall prey to more complex "double-iO" attacks:

  Run one obfuscated circuit on the description of **another** obfuscated circuit.

Final Thoughts

- UCEs bring added provable-security guarantees to practical protocols.
- Progress 15 years after [CGH98].
- This work brings cryptanalytic validation & refinements to UCEs:
  - A natural computational UCE notion is uninstantiable.
  - Our weaker statistical UCE is robust and still useful.
- iO attack extends to many prominent ROM transforms:
  - D-PKE, Hedged PKE, Fujisaki–Okamoto, KDM, MLEs, ...

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- Characterize the power of iO attacks more precisely.
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Thank you.