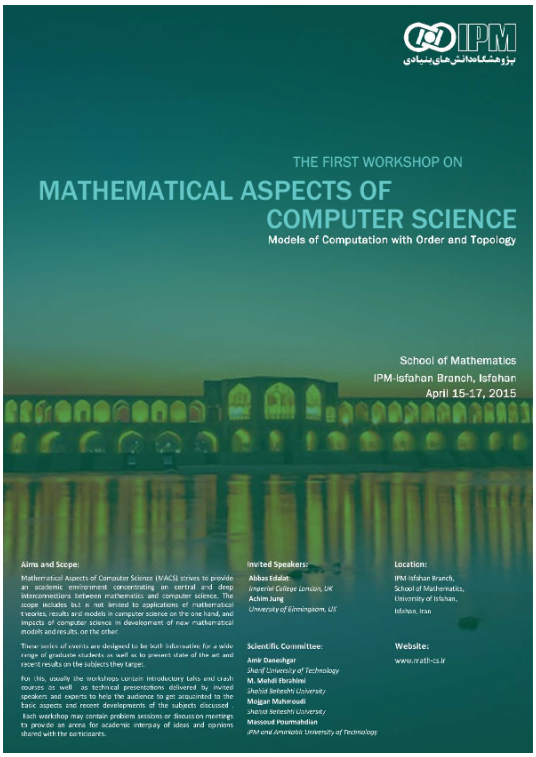
Abstracts & Program

1st Workshop on Mathematical Aspects of Computer Sciences:

Models of Computation with Order and Topology



15-17 April, 2015
IPM-Isfahan Branch, University of Isfahan, Isfahan, Iran
www.math-cs.ir

Introduction

Mathematical Aspects of Computer Science (MACS) strives to provide an academic environment concentrating on central and deep interconnections between mathematics and computer science. The scope includes but is not limited to applications of mathematical theories, results and models in computer science on the one hand, and impacts of computer science in development of new mathematical models and results, on the other.

This workshop is the first workshop of a series of events which are designed to be both informative for a wide range of graduate students as well as to present state of the art and recent results on the mathematical aspects of computer science. The main subject of this workshop is models of computation with order and topology.

The workshop contains introductory talks and crash courses as well as technical presentations delivered by invited speakers and experts to help the audience to get acquainted to the basic aspects and recent developments of the subjects discussed. Moreover, some related contributed talks have been accepted to be presented.

Scientific Committee

Program

	Wednesday April 15th, 2015	Thursday April 16th, 2015	Friday April 17th, 2015
	9-9:10 opening		
9:10-10:20	Programs, domains, and logic **Achim Jung**	Very Elementary domain theory: The origins of the subject via lambda calculus **Saleh Aliyari**	On domain algebras **Achim Jung**
10:20-10:50	Break	Break	Break
10:50-12	Domain theory: a framework for non-Hausdorff model theory **Massoud Pourmahdian**	The metamathematics of domain theory and beyond **Amir Daneshgar**	Synthetic domain theory for $\mathbb{N}^{\infty}$-sets **Mojgan Mahmoudi**
12-14	Lunch	Lunch	Closing-Lunch
14-15:10	Domain theory and differential calculus **Abbas Edalat**	Domain theory and Integral calculus **Abbas Edalat**	
15:15-15:45	Break	Break	
15:45-16:15	Congruences and isomorphism theorem on directed complete partially ordered sets **Halimeh Moghbeli**	Computable analysis and its applications **Nazanin Roshandel Tavana**	
16:20-16:50	Injectivity and retractness of directed complete poset acts **Mahdieh Yavari**	**Discussion**	
16:50:17:40			

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Organizing Committee[1]

Javad Asadollahi | Mojgan Mahmoudi | Massoud Pourmahdian

Scientific Committee

Amir Daneshgar | M. Mehdi Ebrahimi
Mojgan Mahmoudi | Massoud Pourmahdian

Invited Speakers

Abbas Edalat | Achim Jung | Saleh Aliyari

Lectures

- **Saleh Aliyari**
  Very elementary domain theory: The origins of the subject via lambda calculus
- **Amir Daneshgar**
  The metamathematics of domain theory and beyond
- **Abbas Edalat**
  1- Domain theory and integral calculus
  2- Domain theory and differential calculus
- **Achim Jung**
  1- Programs, domains, and logic
  2- On domain algebras
- **Mojgan Mahmoudi**
  Synthetic domain theory for $\mathbb{N}^{\infty}$-sets
- **Massoud Pourmahdian**
  Domain theory: a framework for non-Hausdorff model theory

[1]Thanks to the helpfull efforts of Mr Meysam Madani, Mr Mohsen Rahpeyma, and Ms Baran Yazdi.

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Very Elementary Domain theory: The Origins of the Subject via Lambda Calculus

Saleh Aliyari
IPM
Tehran, Iran

Abstract

What are domains? Why and how were they first studied? In this elementary, semi-historical talk we study the body of work that was almost entirely done by one man, Dana Scott. Scott worked on developing a model for untyped lambda calculus which was in a way a formalism of computation. The problem was that there was no semantics developed for it. The reason was that functions over a set (say of natural numbers) are represented in lambda calculus and since there are no types there is no syntactic distinction between functions of mere "natural numbers" and functions of functions of natural numbers. So there was no hope for a set theoretical model because of the inherent distinction between a collection of sets and collections of functions between those sets. Scott developed an order theoretic topology that made it possible for Lambda terms to be represented as elements of a "domain", the Scott domain. Lambda calculus can be seen as a very simple programming language. So in a way Scott's work was the first step in the semantics of programming languages as we know today and in particular of domain theory. Very complex domains are studied nowadays and there is a lot of depth to the subject. The goal of this talk is to give a modest view of the very beginning of this very active subject matter. We do not assume anything but some familiarity with basic topology and the talk is presented in an intuitive manner.

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The Metamathematics of Domain Theory and Beyond

Amir Daneshgar
Department of Mathematics
Sharif University of Technology, Tehran, Iran

Abstract

The main objective of this talk is to show that *denotational semantics* can be considered within the context of *categorical system theory* as *computation* can be considered within the context of *discrete dynamics*, which definitely pins the subject at the center of the universe of mathematical sciences.

In this regard, considering the fixed points of a functor as stable points of a feedbacked dynamics, we will discuss the main problems of nonlinear categorical system theory. Following this, and referring to a result of K. R. Wagner (1994-7), we will try to show that one of the central concepts to be focused on is a categorification of the topology of order convergence in an enriched sense.

The rest of the talk will be about how this categorification might be used to build an extension of the theory of accessible categories as a unifying framework for the subjects proposed.

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Domain Theory and Differential Calculus

Abbas Edalat
Department of Computing
Imperial College London, UK

Abstract

We introduce the notion of a partial extension of an arbitrary, not necessarily continuous, map from a subset of a topological space into a continuous Scott domain and deduce a universal property for such a domain with respect to partial maps, extending the densely injective property of continuous Scott domains. It is then proved that the Clarke gradient of a Lipschitz map between two finite dimensional real Euclidean spaces is the partial extension of the classical derivative of the Lipschitz map, which exists on a dense subset. We then show that the subspace of real-valued continuously differentiable functions on a finite dimensional Euclidean space is dense in the space of Lipschitz maps equipped with the L-topology. This provides a new result in basic mathematical analysis, which characterises the L-topology in terms of the limsup of the the sequence of derivatives of a sequence of C1 maps that coverages to a Lipschitz map. Using this result, we finally prove that the generalised (Clarke) gradient on Lipschitz maps is the extension of the derivative operator on maps.

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Domain Theory and Integral Calculus

Abbas Edalat
Department of Computing
Imperial College London, UK

Abstract

We show that the generalised Riemann integral (R-integral) of a real-valued continuous function on a compact metric space with respect to a Borel measure can be extended to the integral of interval-valued functions on the metric space with respect to valuations on the probabilistic power domain of the space of non-empty and compact sets of the metric space. This extension is precisely the extension obtained by the densely injective property of continuous Scott domains. It provides a new method of computing the integral in which both the function and the Borel measure can be approximated by finitary objects, namely step functions and simple valuations, respectively. We also prove that the Lebesgue integral operator on integrable functions is the extension of the R-integral operator on continuous functions. This provides a new way of computing the Lebesgue integral using the R-integral. We finally illustrate an application of these results by deriving a simple proof of Green's theorem for interval-valued vector fields.

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Programs, Domains, and Logic

Achim Jung
School of Computer Science
The University of Birmingham, Birmingham, UK

Abstract

In this talk I will give an overview of some research questions that over the years have inspired me and to which I have contributed. While somewhat tutorial in nature, I will also try to explain some of the more technical difficulties which we have encountered along the way, and which have made me change direction a few times.

We will begin with Scott's work of the late 1960's, when he began to lay the foundations for a "mathematical theory of computation", and the correspondence between programs and semantic models initiated by Plotkin in 1977. I will explain the notion of a Scott domain and its associated concepts of "finite element" and "approximation". We'll then see how the desire to deal with real numbers and probabilities led to an interest in continuous domains. Again we'll see its successes but also some of the problems that appeared. For reasons of the latter, the focus for me shifted towards topological spaces. Domain-style approximation being no longer available, this move entailed a closer study of Stone dualities, where instead of spaces one considers "propositions" about elements of a space. Although this may appear as a technical necessity, it does actually emphasise the link between semantics and program logics, something that Samson Abramsky advocated more than 20 years ago in his "Domain theory in logical form".

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On Domain Algebras

Achim Jung
School of Computer Science
The University of Birmingham, Birmingham, UK

Abstract

Domains (in the sense of Dana Scott) can be characterised via their order structure or via their (Scott) topology. A lot is known about their structural properties both at the individual and the categorical level. In applications, in addition to the order structure some algebraic operations satisfying certain equations are often required. The interplay between order/topology and algebra is not entirely straightforward but we do now have some general existence theorems.

In this talk I intend to present the various approaches to domain theory that one might be interested in from an applications point of view, and then explain the difficulties one has to overcome if one is trying to add algebraic structure. In joint work with M. A. Moshier and S. Vickers, we have found a new way of constructing domain algebras which is much more concrete than the method that was employed before. More recently still, K. Keimel and J. Lawson discovered that this can be explained and extended very elegantly by replacing order with topology.

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Synthetic Domain Theory for $\mathbb{N}^{\infty}$-sets

Mojgan Mahmoudi[2]
Department of Mathematics
Shahid Beheshti University, Tehran, Iran

Abstract

Following Dana Scott's idea of studying domain-theoretic structures as set-theoretic structures in a model of intuitionistic set theory; especially thinking "domains are sets" and "continuous maps are maps" and working in a topos, the idea of synthetic domain theory (SDT) formalized in 1991 by Hyland, also Taylor. Then followed and expanded by some other people like Rosolini, Phoa, and Simpson, also few models of synthetic domain theory were introduced (see for example Fiore's paper).

In this talk, first the foundations of Synthetic Domain Theory are presented. Then we consider this theory in the presheaf topos of $\mathbb{N}^{\infty}$-sets, where $\mathbb{N}^{\infty}$ in the monoid of extended natural numbers with the minimum as the binary operation.

[2]joint with Achim Jung

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Domain Theory: A framework for non-Hausdorff model theory

Massoud Pourmahdian
School of Mathematics
IPM and Amirkabir University of Technology, Tehran, Iran

Abstract

In this talk I will present a model-theoretic frame for studying continuous domains endowed with some extra structures. A suitable logic is developed and by inspiring from continuous model theory certain interpretations for existential quantifier is suggested. Some model-theoretic constructions, such as elementary chains and ultraproducts are adopted in this context, enabling us to prove that the class of continuous domain structures together with the corresponding notion of elementary embeddings forms an abstract elementary class. Finally, the spaces of types are defined and it is shown that these spaces are T_1 and compact and furthermore, they satisfy some additional nice domain-theoretic properties.

Contributed Talks

- **Halimeh Moghbeli**
  Congruences and isomorphism theorem on directed complete partially ordered sets
- **Nazanin Roshandel Tavana**
  Computable analysis and its applications
- **Mahdieh Yavari**
  Injectivity and retractness of directed complete poset acts

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Congruences and Isomorphism Theorem on Directed Complete Partially Ordered Sets

Halimeh Moghbeli[3]
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University of Jiroft, Jiroft, Iran

Abstract

Directed complete partially ordered sets (briefly dcpo's) play an important role in domain theory. The first aim of this paper is to describe congruences of dcpo's. We also show that the kernel of dcpo maps, that is, directed join-preserving maps between dcpo's are not necessarily a dcpo congruence. Therefore we characterize dcpo maps whose kernels are dcpo congruences. Finally, we prove the Decomposition and Isomorphism Theorems for dcpo maps.

[3]joint with M. Mahmoudi and K. Pioro

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Computable Analysis and its Applications

Nazanin Roshandel Tavana
Amirkabir University of Technology and IPM, Tehran, Iran

Abstract

Computable analysis is a branch of computability theory studying those real functions and the related sets which can be computed by ma- chines such as digital computers. The increasing demand for reliable soft- ware in scienti

c computation and engineering requires a sound foundation not only of the analytic/numerical but also of the computational aspects of real number computation. The central subject of this talk is one of the approch of computable analysis called "Type Two Theory of Effectivity (TTE)". It is based on definitions of computable real numbers and func- tions by A. Turing, A. Grzegorczyk and D. Lacombe. First, computability on finite and infinite sequences of symbols is introduced. Then, this kind of computability can be transfered to the other sets by means of names or codes. After, the framework of computability is settled down, we can talk about the effectiveness of some other spaces as metric spaces. At the end, complexity of this approach and its two applications are discussed. One application is for effectiveness in metric model theory and the other one is in measure theory.

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Injectivity and Retractness of Directed Complete Poset Acts

Mahdieh Yavari[4]
Department of Mathematics
Shahid Beheshti University, Tehran, Iran

Abstract

Action of a monoid or a group on a set have always been of interest for mathematicians and computer scientists. On the other hand, domain theory, which studies directed complete partially ordered sets, was introduced by Scott in the 1970s as a foundation for programming semantics and provides an abstract model of computation, and has grown into a respected field on the borderline between mathematics and computer science.

In this talk, combining the above two notions, we consider actions of a semigroup (monoid or group) on directed complete posets and study the algebraic notions of injectivity and retractness with respect to monomorphisms and embeddings in the category so obtained.

[4]joint with M.M. Ebrahimi and M. Mahmoudi